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METHODS OF OPTIMAL DESIGNING OF CONSTRUCTIONS OF CONVENTIONAL AND COMPOSITE MATERIALS SUBJECTED TO WAVE AND STATIC ACTIONS

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The problem of optimal designing of inhomogeneous constructions with a combination of properties prescribed in advance is considered. The varied parameters include all the parameters determining the structure of an optimal construction: the physical properties of the layers, the geometric thicknesses of the layers, and the number of layers. An efficient procedure of optimal designing has been developed. The problem of synthesis of the optimal geometry of a stiffened shell with simultaneous search for its optimal structural design has been solved. The mass of the stiffened shell is minimized with allowance for the restrictions of fracture mechanics and static strength and also the restrictions for the stiffness characteristics of the shell in bending and torsion. The method developed and the computational program can be used in designing stiffened shells.

Optimal Designing of Inhomogeneous Constructions. In the last decades, increasing attention has been given to the problem of optimal designing of composite constructions that interact with wave fields of different physical nature (electromagnetic, acoustic, thermal, elastic).

The problems of optimal designing of composite constructions with layers having different physical properties have been investigated. The varied parameters are the physical properties of the materials of the layers, the thicknesses of the layers, and the number and relative position of the layers. For electromagnetic waves, the physical properties are the permittivity and the permeability of the layers. For acoustic waves, these are the density and the velocity of their propagation in the layers. For an elastic wave, the physical properties are the density of the layers and the velocities of propagation of longitudinal and transverse waves. For heat waves, these are the thermal-conductivity and thermal-diffusivity coefficients of the layers.

Of primary importance is the problem of investigation of the limiting capabilities of composite constructions for attaining a prescribed combination of properties. The situation where a designer has a discrete set of materials that can be used in designing is the closest to actual designing. Because of this, part of the parameters of a construction (physical properties of the materials of the layers) can take on only discrete values.

We have given the formulation of the variational problem of optimal designing of a composite construction with a combination of properties prescribed in advance. Such problems of designing inhomogeneous structures subjected to wave actions of different physical nature have been formulated in uniform form as problems of optimal control of complex systems with a hierarchical structure.

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The problem of optimal designing of an inhomogeneous construction can be formulated in general form as follows:

$$Lu = g\,, \tag{1}$$

$$u \in U\,, \tag{2}$$

$$J(u) = \|Lu - \tilde{g}\|^2 \to \min_{u \in U}\,. \tag{3}$$

Here (1) is the operator equation that relates the structure of the composite construction to the energy characteristics of the wave processes which are different in physical nature (electromagnetic, acoustic, thermal, elastic) and interact with the composite construction. Knowledge of the structure of the composite construction allows one to determine the energy characteristics of the wave processes. Function (3) characterizes the criterion of proximity of the energy characteristics of a wave process g to the required characteristics $\tilde{g}$. The problem of optimal designing implies that it is necessary to find all the permissible variants of the construction U^* which provide the global minimum of the function

$$U^* = \left\{ u^* \in U : \|Lu^* - \tilde{g}\|^2 = \min_{u \in U} \|Lu - \tilde{g}\|^2 \right\}. \tag{4}$$

The elements of the set U^* realize the limiting capabilities of composite constructions for attaining a prescribed combination of properties.

The existing approaches to solution of the problems of optimal designing are associated, as a rule, with a simplified formulation where part of the control parameters determining the structure of a composite construction are fixed. As a rule, the varied parameters are only the thicknesses of the layers of the construction, which allows one to reduce the problems of optimal designing to special problems of nonlinear programming, where solution involves minimization of the objective function on a set determined by the restrictions of the problem. Then, the formulated problem is solved using the existing methods of nonlinear programming. However, this approach to solution of the problems of optimal design of an inhomogeneous construction with a combination of properties prescribed in advance, which is associated with a significant simplification of the initial problem, gives no way to investigate in full measure the limiting capabilities that can be realized by direct exhaustion of the variants of construction.

The complete solution of the problem formulated means that the possibility of exhausting efficiently and completely all the admissible variants of constructions exists. Since such a number of all permissible variants of constructions is very large in the problems of optimal design, realization of the methods of complete exhaustion is impossible even with the use of highly efficient computers.

We have developed new methods for a detailed investigation of the limiting capabilities of inhomogeneous constructions, which do not require a complete exhaustion of all the variants.

In accordance with the posed problem of optimal designing of inhomogeneous constructions, we have developed the necessary optimality conditions associated with nonlocal variations of the control parameters. On this basis, we have proposed computational procedures of optimization which allow one to efficiently find all the sought parameters determining the structure of a composite construction: the physical properties of materials, the thicknesses of the layers, and the number and relative position of the layers with different physical properties. The discrete domain of the values of a number of parameters determining the physical properties of the materials of the layers can also be established efficiently.

We have revealed new qualitative laws of composite constructions, which make it possible to realize the limiting capabilities for attaining the prescribed combination of properties [1–3, 6–9].

The use of the developed methods of optimal designing of inhomogeneous constructions with a combination of properties prescribed in advance opens up promising new opportunities for solving problems in this field.

Optimal Designing of Stiffened Shells. In connection with the constantly increasing requirements placed on an aircraft, the choice of its optimal design becomes more and more important. A combination of opposite restrictions that requires a compromise between a high weight efficiency of the construction, on the one hand, and a long service life, on the other, leads one to the necessity of considering the issues of fatigue and failure. In preliminary designing, simplified approaches of a fatigue analysis (such as setting the limit of permissible stresses and others) are used as a rule. The more exacting requirements on the weight of an aircraft and its reliability gave birth to the concept of a safely damaged construction. This concept allows a safe growth of fatigue cracks up to the point of their visual detection. Optimization of such a construction is a more complex problem for the following reasons: it is necessary to determine the residual strength after cracking and the stress intensity factor at the apices of standard longitudinal and transverse cracks and also develop a method of nonlinear mathematical analysis for solving this problem in the appropriate time with the highest degree of accuracy.

Cylindrical hermetic shells loaded by bending moments, forces acting in the cross-sectional plane of a shell, and the internal pressure are considered. The shells consist of cylindrical panels (the neighboring panels have a common tangent). The optimal design is chosen among stiffened shells in which none of the portions of the casing subjected to the action of design loads has a local stability loss [4, 5, 10]. To determine the normal and shear stresses in the absence of cracks, the classical beam theory of thin-walled stiffened shells is used [11]. The total stability loss in bending is estimated in the same manner as for a structurally anisotropic shell [12], and the flattening of its cross section in bending is determined from a semiempirical formula [13]. The local loss of stability of structural elements is taken into account on the basis of the dependences approved repeatedly in the practice of creation of constructions [12, 14]. Theoretical dependences corrected on the basis of experiments are used to establish the residual strength in the presence of cracks and to determine the stress intensity factors at the apices of the standard longitudinal and transverse cracks or holes in the upper, side, and lower panels [15, 16].

The method developed makes it possible to synthesize the optimal geometry of a hermetic shell with variation of the shape of the cross section and simultaneously determine its optimal structural design with variation of the number of primary elements and their size.

The independent parameters of optimal designing comprise 17 quantities. For the upper panel, the two identical side panels, and the lower panel they are, respectively, the thicknesses of the casings, the cross-sectional areas, and the number of longitudinal stiffening elements in each panel. The optimized parameters also include the cross-sectional areas, the number of transverse stiffening elements, and the quantities determining the shape of the contour of the cross section of a hermetic shell, i.e., the height and width of the cross section of the contour and the coordinates of the points of smooth remating (with the common tangent) of the panels.

Minimization of the mass of a hermetic shell is carried out in the presence of functional restrictions in the form of inequalities:

$$f_j(x_i) > 0 . \tag{5}$$

Optimization of the construction under consideration is carried out with allowance for the following main restrictions: on static strength under design loads, on fracture mechanics for possible fatigue cracks in a hermetic shell under a maximum operating load and in the case of suddenly arising holes, and on the stiffness characteristics of a hermetic shell in its lateral bending and torsion. We can also consider the restrictions on the frequencies of natural vibrations of the portions of the panel casing. In designing, these restrictions allow one to monitor the incipient fatigue cracks in the casing and track the level of noise penetrating into the construction. The restrictions on the dimensions of the cross section (width and height) and the restrictions on the minimum and maximum permissible values of the parameters of the optimal design are considered.

According to the method of internal penalty logarithmic functions, with restrictions (5), the problem of minimization of the mass of a hermetic shell is reduced to the problem of unconditional minimization of the auxiliary objective function

$$F = M - A_k \sum_j \ln f_j(x_i) = \min, \quad A_k > 0, \quad A_{k+1} > A_k, \lim_{k\to\infty} A_k \to 0. \tag{6}$$

Minimization of the auxiliary objective function is carried out by the method of coordinatewise descent with cyclic exhaustion of variables at each step and with a varied step in each variable. A search for the solution consists of two cycles: the internal cycle (for a fixed A_k) and the external cycle (for a varied A_k).

The number of steps of the internal and external cycles is determined in the process of calculations. Regulation of the variation step in each variable is carried out with the use of the Fibonacci sequence of numbers. This method was used for the first time in [4] for the nonlinear problem of optimization with many variables.

We have developed a program for optimizing the forms of hermetic shells having a minimum mass and the positions and dimensions of their stiffeners with allowance for different reqirements: static strength, stiffness, service life, operating survivability, and dimension and design restrictions.

The program is meant for designers, arrangers, technologists, specialists in materials science, and specialists in fatigue strength and in investigation of the operating survivability of constructions with cracks and holes. The use of the program makes it possible to improve the efficiency of cooperation in development of perfect hermetic shells, which is of prime importance at the initial stage of designing. The program, playing the role of a supervisor between specialists in different fields, allows one to:

(1) minimize the mass of the compartment of a hermetic shell;

(2) select the optimal structural material;

(3) find the optimal geometry of the cross section;

(4) determine the optimal position of the junctions between the panels;

(5) find the most suitable distance between the longitudinal and transverse stiffening elements;

(6) take into account about one hundred restrictions in the form of inequalities in optimization: (a) on the stability, the static strength, and the stiffness characteristics of a hermetic shell; (b) on the minimum permissible size and classes of the stringers and transverse frames used; (c) on safe operation in the presence of "standard" cracks and holes in a hermetic shell; (d) on the outside and inside dimensions of a hermetic shell.

The program also allows one to check the observance of all the restrictions in the constructions designed and allows the addition of any new restrictions in the form of inequalities to it.

Using the algorithms developed and the computational program, we have calculated the compartments of hermetic fuselages of three types (of two passenger airplanes and one maneuver airplane). The mass of these compartments was found to be 9–14% smaller than the mass of compartments thoroughly designed by conventional methods in three different design offices.

NOTATION

L, designing operator; J, figure of merit; u, element of the set of structures of an inhomogeneous construction; U, set of permissible variants of structures of the construction; g, functional element characterizing the energy characteristics of the wave process; $\tilde{g}$, required energy characteristics of the wave process;

u^*, element of the set of variants of constructions which provide a global minimum; U^*, all the permissible variants of constructions which provide a global minimum; x_i, independent parameters of optimal designing; f_j, functional restrictions of optimal designing; F, auxiliary objective function; M, mass of the construction; A_k, descending sequence of coefficients of the objective function. Subscripts: i, numbers of independent parameters of optimal designing; j, numbers of functional restrictions of optimal designing; k, numbers of the elements of the sequence of coefficients of the objective function.

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